

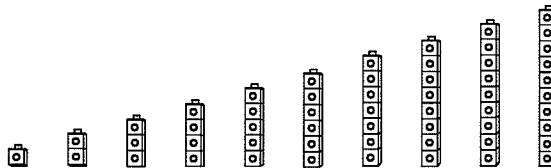
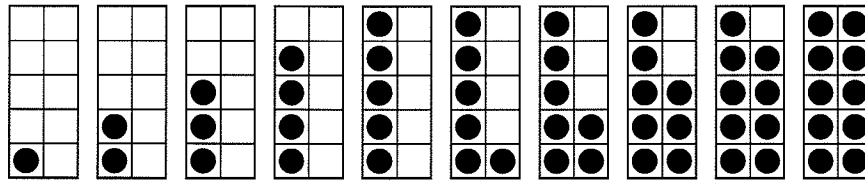
NUMBER FACTS TO 20

Recognizing numbers as one ten and some more

What is essential to learning the basic addition and subtraction facts to 20:

- To know how many are needed to make ten
- To combine quantities by reorganizing into one ten and leftovers (ones) without counting
- To know ten plus any number from one to ten
- To tell how many leftovers when removing ten for numbers 11 to 20
- Subtract quantities by breaking numbers apart and recombining whatever is left

Materials: White boards or scratch paper; Ten frames; Interlocking cubes; Digit cards



Use this packet if students need to learn:

- Ten plus a number without counting
- Nine plus a number without counting
- Eight plus a number without counting
- Seven plus a number without counting
- Six plus a number without counting
- A number minus ten without counting
- Differences of ten without counting
- A number minus 9 without counting
- A number minus 8, 7, 6, 5, 4, 3, without counting

MEANINGFUL FLASHCARDS provide opportunities for students to work with number facts between 10 and 20 in ways that give them access to the facts without using their fingers.

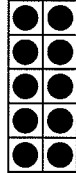
The teacher presents various problems to students and asks them to share the processes they used to figure out “how many.”

Ten Plus a Number

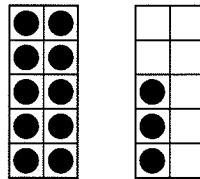
TEN PLUS A NUMBER: MEANINGFUL FLASHCARDS

- Show a ten frame with ten dots and ask, “How many do you see?”

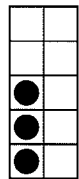
For example:



- Show the second card and ask, “How many do you see?” For example:
- Put the two cards together and ask, “How many do you see now?” For example,



- Students explain their way of combining the dots.
- Record the problem horizontally for students to see ($10 + 3 = 13$; or $3 + 10 = 13$; or $13 = 10 + 3$; or $13 = 3 + 10$).
- Continue until students instantly see amounts **without counting**.



TEN PLUS A NUMBER: NUMBER TRAINS

- Show a ten train and ask,

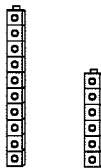
“How many do you see?” For example: 

- Show a second number train and ask,

“How many do you see?” For example:



- Put the two trains together and ask, “How many do you see now?”
For example,

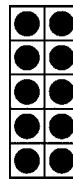


- Students explain their way of combining the cubes
- Record the problem horizontally for students to see ($10 + 6 = 16$; or $6 + 10 = 16$; or $16 = 10 + 6$; or $16 = 6 + 10$).
- Continue until students instantly see amounts **without counting**.

TEN PLUS A NUMBER: WHAT IF?

- After students instantly recognize ten plus a number with ten frames, continue the work with the following tasks.

- Show students a ten frame with ten dots.



- Ask, “What if we added (1, 2, 3, 4, 5, 6, 7, 8, 9, or 10) more dots? How many would there be?”

- Record the problem horizontally for students to see ($10 + 6 = 16$; $16 = 10 + 6$)
- Continue until students instantly recognize the amounts without counting.

TEN PLUS A NUMBER: WHAT IF?

- After students instantly recognize ten plus a number with number trains, continue the work with the following tasks.
- Show students a ten train.
- Ask, “*What if we added (1, 2, 3, 4, 5, 6, 7, 8, 9, or 10) more cubes? How many would there be?*”
- Record the problem horizontally for students to see ($10 + 6 = 16$; $16 = 10 + 6$)
- Continue until students instantly recognize the amounts without counting.



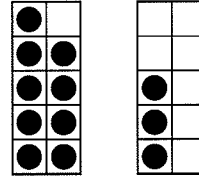
TEN PLUS A NUMBER: SYMBOLS

- Show students the ten digit card (or ten train) and a digit card of another amount (1, 2, 3, 4, 5, 6, 7, 8, 9, 10).
- 10** **4**
- As you show each digit card ask, “*What if we had ten dots (or cubes) and added (1, 2, 3, 4, 5, 6, 7, 8, 9) more dots (or cubes). How many would there be?*”
 - Record the problem horizontally for students to see ($10 + 4 = 14$; $14 = 10 + 4$)

Nine Plus a Number

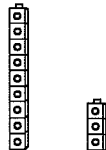
NINE PLUS A NUMBER: MEANINGFUL FLASHCARDS

- Show students ten frame with nine dots paired with another ten frame of less than ten dots for three seconds. For example:
- Ask, "How many do you see?"
- Ask, "How did you see it?"
- Students explain their strategies for combining the dots.
- Continue until students instantly see amounts without counting.



NINE PLUS A NUMBER: NUMBER TRAINS

- Show students a nine train paired with another train of less than ten cubes for three seconds.
- For example:
- Ask, "How many do you see?"
 - Ask, "How did you see it?"
 - Students explain their strategies for combining the cubes
 - Continue until students instantly see amounts without counting.



IMPORTANT!!! PLEASE READ

MAKE A TEN STRATEGY

THE STRATEGIES THAT WE ARE HOPING THAT STUDENTS UNDERSTAND ARE THOSE THAT INVOLVE TEN. FOR EXAMPLE:

“I TOOK ONE FROM THE THREE AND PUT IT WITH THE NINE AND THAT MAKES TEN. I NOW HAVE TEN AND TWO AND THAT MAKES TWELVE.”

IF STUDENTS DO NOT JUST KNOW AND DO NOT USE THE **MAKE A TEN STRATEGY** (EVEN IF THEY HAVE THE CORRECT ANSWER), THE TEACHER ASKS:

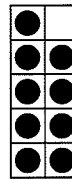
“How many would you need to make a ten? (1)

“How many leftovers would there be?” (2)

“How many is that altogether? (12)

NINE PLUS A NUMBER: WHAT IF?

- After students instantly recognize nine plus a number with ten frames, continue the work with the following tasks.
- Show students a ten frame with nine dots.
- Ask students, “What if we added (1, 2, 3, 4, 5, 6, 7, 8, 9) more dots. How many would there be?”
- If students have difficulty, ask:
 - “How many would you need to make a ten?”
 - “How many leftovers would there be?”
 - “How many would this be altogether?”
- Record the problem horizontally for students to see ($9 + 6 = 15$; $15 = 9 + 6$)
- Continue until students instantly recognize the amounts without counting.



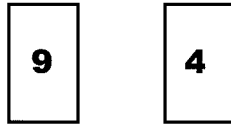
NINE PLUS A NUMBER: WHAT IF?

- After students instantly recognize nine plus a number with number trains, continue the work with the following tasks.
- Show students a nine train.
- Ask students, “What if we added (1, 2, 3, 4, 5, 6, 7, 8, 9) more cubes. How many would there be?”
- If students have difficulty, ask:
 - “How many would you need to make a ten?”
 - “How many leftovers would there be?”
 - “How many would this be altogether?”
- Record the problem horizontally for students to see ($9 + 6 = 15$; $15 = 9 + 6$)
- Continue until students instantly recognize the amounts without counting.



NINE PLUS A NUMBER: SYMBOLS

- Show students the nine digit card (or nine train) and a digit card of another amount.

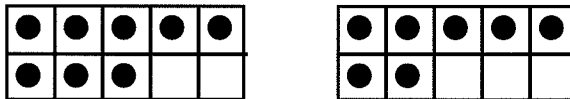


- As you show the students each digit card ask, *“What if we had nine dots (or cubes) and added (1, 2, 3, 4, 5, 6, 7, 8, 9) more dots (or cubes). How many would there be?”*
- If students have difficulty, ask:
“How many would you need to make a ten?”
“How many leftovers would there be?”
“How many would this be altogether?”
- Show the two problems $9 + 4 = 13$ and $4 + 9 = 13$
- Ask, *“Explain to me why these two equations have the same sum?”*
Students use the ten frames or the number trains to make their argument.

Eight Plus a Number

EIGHT PLUS A NUMBER: MEANINGFUL FLASHCARDS

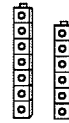
- Show students ten frame with eight dots paired with another ten frame of less than ten dots for three seconds. For example:



- Ask, *“How many do you see?”*
- Ask, *“How many would you need to make a ten?”* (2)
- Ask, *“How many leftovers would there would be?”* (5)
- Ask, *“How many would this be altogether?”* (15)
- Record the problem horizontally for students to see ($8 + 7 = 15$; or $7 + 8 = 15$; or $15 = 8 + 7$; or $15 = 7 + 8$)
- Continue until students instantly see amounts without counting.

EIGHT PLUS A NUMBER: NUMBER TRAINS


- Show students an eight train paired with another train of less than ten cubes for three seconds.
For example:
- Ask, "How many do you see?"
- Ask, "How many would you need to make a ten?" (2)
- Ask, "How many leftovers would there be?" (5)
- Ask, "How many would this be altogether?" (15)
- Record the problem horizontally for students to see ($8 + 7 = 15$; or $7 + 8 = 15$; or $15 = 8 + 7$; or $15 = 7 + 8$)
- Continue until students instantly see amounts without counting.



EIGHT PLUS A NUMBER: WHAT IF?

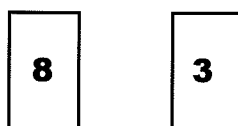
- After students instantly recognize eight plus a number with ten frames, continue the work with the following tasks.
 - Show students a ten frame with eight dots.
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| ● | ● | ● | | |
- Ask, "What if we added (2, 3, 4, 5, 6, 7, 8, 9) more dots. How many would there be?"
 - If students have difficulty, ask:
 - "How many would you need to make a ten?"
 - "How many leftovers would there be?"
 - "How many would this be altogether?"
 - Record the problem horizontally for students to see ($8 + 6 = 14$; $14 = 8 + 6$)
 - Continue until students instantly recognize the amounts without counting.

EIGHT PLUS A NUMBER: WHAT IF?

- After students instantly recognize eight plus a number with number trains, continue the work with the following tasks.
- Show students an eight train 
- Ask, "What if we added (2, 3, 4, 5, 6, 7, 8, 9) more cubes. How many would there be?"
- Ask, "What if we added (2, 3, 4, 5, 6, 7, 8, 9) more cubes. How many would there be?"
- If students have difficulty, ask:
 - "How many would you need to make a ten?"
 - "How many leftovers would there be?"
 - "How many would this be altogether?"
- Record the problem horizontally for students to see ($8 + 6 = 14$; $14 = 8 + 6$)
- Continue until students instantly recognize the amounts without counting.

EIGHT PLUS A NUMBER: SYMBOLS

- Show students the eight digit card and a digit card of another amount.



- As you show the students each digit card ask, "What if we had eight dots (or cubes) and added (2, 3, 4, 5, 6, 7, 8, 9) more dots (or cubes). How many would there be?"
- If students have difficulty, ask:
 - "How many would you need to make a ten?"
 - "How many leftovers would there be?"
 - "How many would this be altogether?"
- Record the problem horizontally for students to see ($8 + 3 = 11$; $3 + 8 = 11$)
- Ask, "Explain to me, why do these two equations have the same sum?" Students use the ten frames or the number trains to make their argument.

SEVEN, SIX, AND FIVE PLUS A NUMBER

- Follow the same strategies as above:

Meaningful Flashcards

Number Trains

What if?

Symbols

A Number Minus 10

A NUMBER MINUS TEN: MEANINGFUL FLASHCARDS

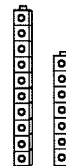
- With ten frames, show students a quantity between 10 and 20. For example:



- Ask, “How many is this altogether?”
- Ask, “How many dots would there be if we took ten away?”
- Record the problem horizontally for students to see ($17 - 10 = 7$; $7 = 17 - 10$).
- Continue until students instantly see amounts without counting.

A NUMBER MINUS TEN: NUMBER TRAINS

- With number trains, show students a quantity between 10 and 20. For example:



- Ask, “How many is this altogether?”
- Ask, “How many cubes would there be if we took ten away?”
- Record the problem horizontally for students to see ($17 - 10 = 7$; $7 = 17 - 10$).
- Continue until students instantly see amounts without counting.

A NUMBER MINUS TEN: WHAT IF?

- Students answer problems by thinking about ten frames and number trains.
- Show students a number card with a number between 10 and 19.

For example:

18

- Ask, "What if we had 18 dots on a ten frame (or 18 cubes) and we took 10 away? How many would be left?"
- Ask students how they thought about the problem to get their answer.
- Record the problem horizontally for students to see ($18 - 10 = 8$; $8 = 18 - 10$).

Differences of Ten

DIFFERENCES OF TEN: MEANINGFUL FLASHCARDS

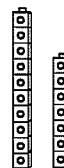
- With ten frames, show students a quantity between 10 and 20. For example:



- Ask, "How many is this altogether?"
- Ask, "How many dots would there be if we took seven away?"
- Record the problem horizontally for students to see ($17 - 7 = 10$; $10 = 17 - 7$).
- Continue until students instantly see amounts without counting.

DIFFERENCES OF TEN: NUMBER TRAINS

- With number trains, show students a quantity between 10 and 20. For example:



- Ask, "How many is this altogether?"
- Ask, "How many dots would there be if we took seven away?"
- Record the problem horizontally for students to see ($17 - 7 = 10$; $10 = 17 - 7$).

DIFFERENCES OF TEN: WHAT IF?

- Show students a number card with a number between 10 and 19.
For example:

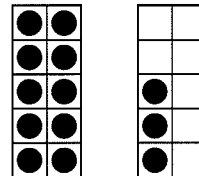
18

- Ask, “*What if we had 18 dots on a ten frame (or 18 cubes) and took 8 away. How many would be left?*”
- Ask students how they thought about the problem to get their answer.
- Record the problem horizontally for students to see ($18 - 8 = 10$; $10 = 18 - 8$).

A Number Minus Nine

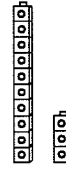
A NUMBER MINUS NINE: MEANINGFUL FLASHCARDS

- Show students ten frames with quantities greater than ten.
- Ask, “*How many dots do you see?*”
- Ask, “*How many dots would there be if we took nine away?*”
- Students explain their strategies for subtracting the dots.
- If students have difficulty, ask:
 - “*What would we need to subtract to make a ten?*” (3)
 - “*How many more do we have to subtract?*” (6)
 - “*How do you know?*” ($6 + 3 = 9$)
 - “*How much is $10 - 6$?*” (4)
 - “*How much is $13 - 9$?*” (4)
- Continue until students instantly see amounts without counting.



A NUMBER MINUS NINE: NUMBER TRAINS

- Show students number trains with quantities greater than ten.
- Ask, "How many cubes do you see?"
- Ask, "How many cubes would there be if we took nine away?"
- Students explain their strategies for subtracting the dots.
- If students have difficulty, ask:
 - "What would we need to subtract to make a ten?" (3)
 - "How many more do we have to subtract?" (6)
 - "How do you know?" ($6 + 3 = 9$)
 - "How much is $10 - 6$?" (4)
 - "How much is $13 - 9$?" (4)
- Students may see that subtracting nine is one less than subtracting ten. Students may subtract the nine and recombine the 1 and 3 to make 4.
- Continue until students instantly see amounts without counting.



A NUMBER MINUS NINE: WHAT IF?

- After students instantly recognize a number minus nine with ten frames (and number trains), continue the work with the following tasks.
- Show students a number card with a number between 10 and 19. For example:

18
- Ask, "What if we had a ten frame (or number train) with (18, 17, 16, 15, 14, 13, 12, 11, 10) dots (or cubes) and we took 9 away? How many would be left?"
- How did you figure it out?
- If students have difficulty, ask:
 - "What would we need to subtract to make a ten?"
 - "How many more do we have to subtract?"
- Record the problem horizontally for students to see.

IMPORTANT!!! PLEASE READ

BREAKING NUMBERS APART STRATEGY AND TENS STRATEGY

STRATEGIES THAT WE HOPE STUDENTS WILL UNDERSTAND ARE THOSE THAT INVOLVE BREAKING NUMBERS APART AND STRATEGIES THAT INVOLVE TENS. FOR EXAMPLE:

“WHAT IF WE HAD A NUMBER TRAIN WITH 17 CUBES AND WE TOOK EIGHT AWAY? HOW MANY WOULD BE LEFT?”

STRATEGIES COULD INCLUDE:

“I BROKE 8 INTO 7 AND 1. THEN I TOOK 7 AWAY FROM 17 AND THAT GAVE ME 10. I HAD 1 MORE TO TAKE AWAY FROM 10 WHICH IS 9, SO $17 - 8 = 9$.”

$$\begin{aligned}8 &= 7 + 1 \\17 - 7 &= 10 \\10 - 1 &= 9\end{aligned}$$

OR

“17 TAKE AWAY 10 IS 7. BUT, I’M NOT TAKING AWAY 10, I’M TAKING AWAY 8, SO I TOOK AWAY 2 TOO MANY AND I HAVE TO ADD 2 TO THE 7 WHICH IS 9, SO $17 - 8 = 9$.”

$$\begin{aligned}17 - 10 &= 7 \\7 + 2 &= 9\end{aligned}$$

OR

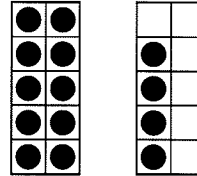
I TOOK 8 AWAY FROM THE TEN. THAT LEFT 2. I ADDED 2 TO THE 7 AND THAT GAVE ME 9.

$$\begin{aligned}17 &= 10 + 7 \\10 - 8 &= 2 \\7 + 2 &= 9\end{aligned}$$

A Number Minus Eight

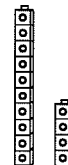
A NUMBER MINUS EIGHT: MEANINGFUL FLASHCARDS

- Show students ten frames with quantities greater than ten.
- Ask, "How many dots do you see?"
- Ask, "How many dots would there be if we took eight away?"
- Students explain their strategies for subtracting the dots.
- If students have difficulty, ask:
 - "What would we need to subtract to make a ten?" (4)
 - "How many more do we have to subtract?" (4)
 - "How do you know?" ($4 + 4 = 8$)
 - "How much is $10 - 4$?" (6)
- "How much is $14 - 8$?" (6)
- Continue until students instantly see amounts without counting.



A NUMBER MINUS EIGHT: NUMBER TRAINS

- Show students number trains with quantities greater than ten.
- Ask, "How many cubes do you see?"
- Ask, "How many cubes would there be if we took eight away?"
- Students explain their strategies for subtracting the dots.
- If students have difficulty, ask:
 - "What would we need to subtract to make a ten?" (4)
 - "How many more do we have to subtract?" (4)
 - "How do you know?" ($4 + 4 = 8$)
 - "How much is $10 - 4$?" (6)
- "How much is $14 - 8$?" (6)
- Continue until students instantly see amounts without counting.



A NUMBER MINUS EIGHT: WHAT IF?

- After students instantly recognize a number minus eight with ten frames, continue the work with the following tasks.
- Show students a number card with a number between 10 and 19.
For example:

15

- Ask, "What if we had a ten frame (or number train) with (17, 16, 15, 14, 13, 12, 11, 10) dots (or cubes) and we took 8 away? How many would be left?"
- How did you figure it out?
- If students have difficulty, ask:
"How many would we need to subtract to make a ten?"
"How many more do we have to subtract?"
- Record the problem horizontally for students to see.
- Continue until students instantly recognize the amounts without counting.

A NUMBER MINUS SEVEN, SIX, FIVE:

- Follow the same strategies as above:

Meaningful Flashcards
Number Trains
What if?

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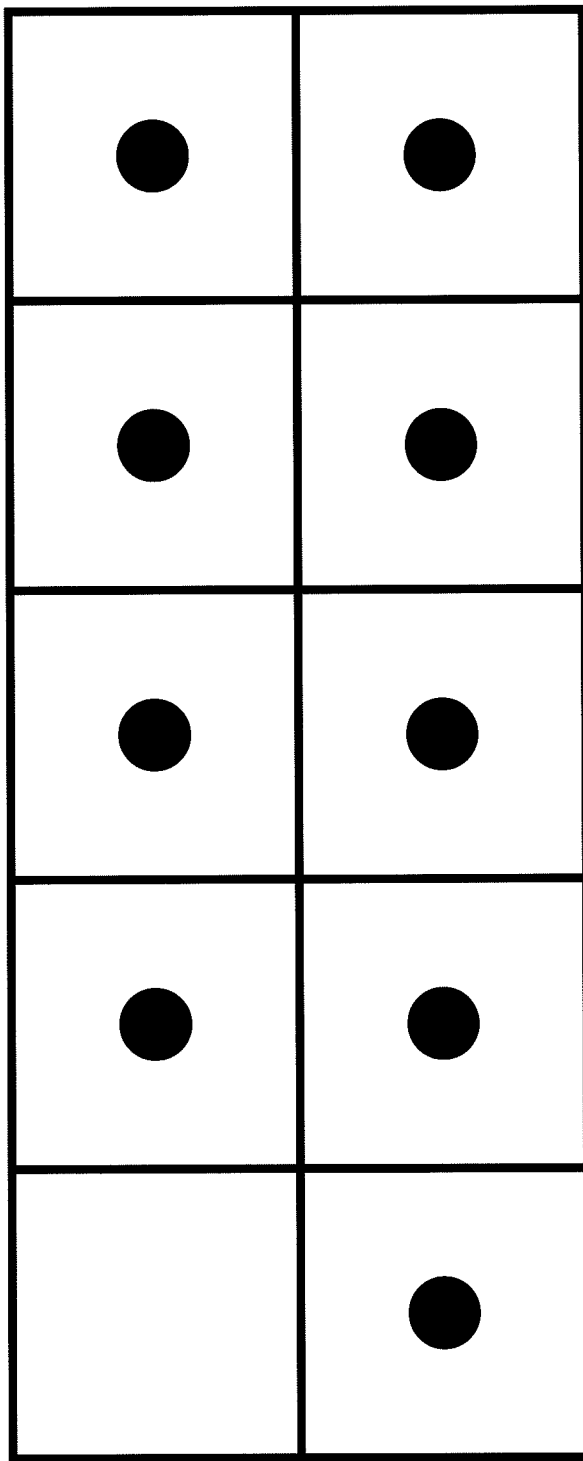
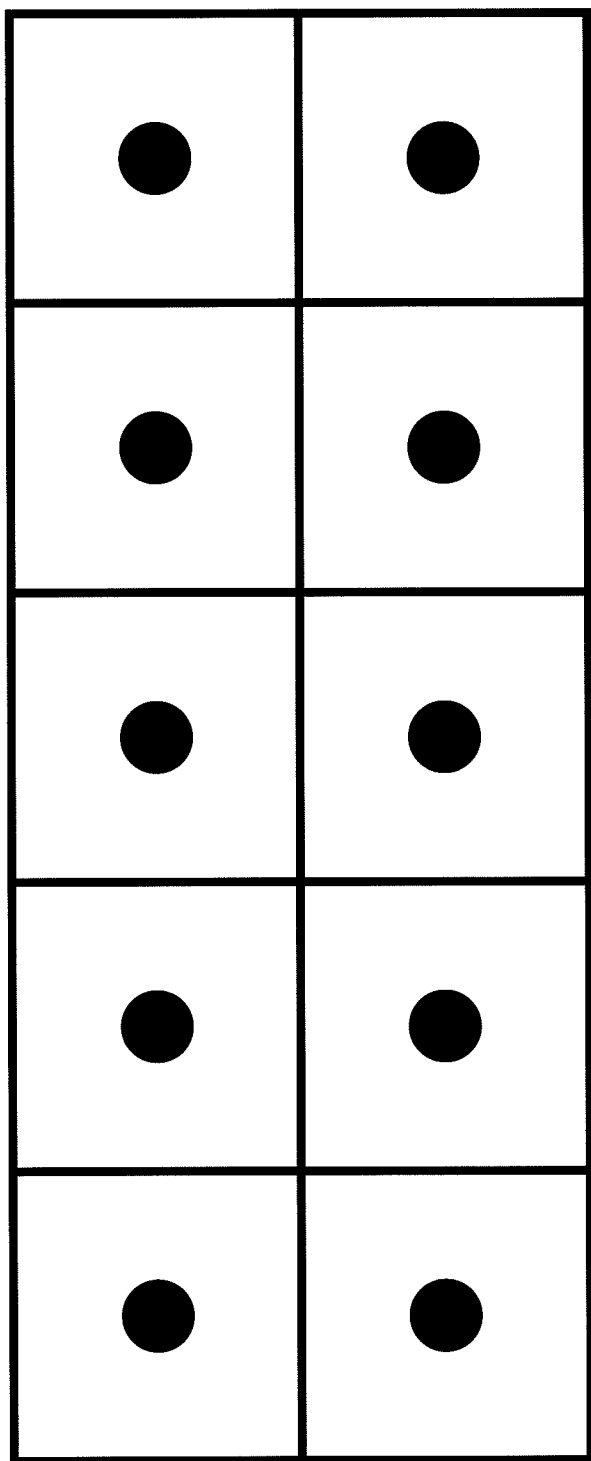
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Digit Cards (cut out)

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17

18	19	20
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